papers essentially economic: witness the numerous interesting papers read to Sections A and G, on the applications of electric energy, and, best of all, the presidential addresses to those sections, of which both were devoted to economic physics, or rather, as the subject should be called, physical economics.

Finally, if the preceding axioms and the soundness of the above classification of social facts be not disproved, it follows that three out of the four great reforms demanded in Mr. Ingram's presidential address,[1] and repeated and enforced in Mr. Grant Duff's, are here introduced into the conduct of economic research, namely, "(1) that the study of the economic phenomena of society ought to be systematically combined with that of the other aspects of social existence; (2) that the exce sive tendency to abstraction and to unreal simplifications should be checked; (3) that the *à priori* deductive method should be changed for the historical; while the fourth, that economic laws and the practical prescriptions founded on these should be conceived and expressed in a less absolute form," would readily also be exemplified if the limits of the present paper permitted reference to generalisation and to practice. Again, it is sufficient to quote Mr. Ingram's concluding proposals to show that these have been substantially adopted. The field of the section should be enlarged so as to comprehend the whole of sociology, "since the economic facts of society . . . cannot be scientifically considered apart, and there is no reason why the researches of Sir Henry Maine or Mr. Spencer should not be as much at home here as those of Mr. Fawcett or Prof. Price. Many of the subjects, too, at present included in the artificial assemblage of heterogeneous inquiries known by the name of anthropology really connect themselves with the laws of social development, and if our section bore the title of the sociological, the studies of Mr. Tylor and Sir John Lubbock would find in it their most appropriate place. I prefer the name sociology to that of social science."[2]

THE PROPER PROPORTIONS OF RESISTANCE IN THE WORKING COILS, THE ELECTRO-MAGNETS, AND THE EXTERNAL CIRCUITS OF DYNAMOS[3]

FOR the electro-magnet;
Let L be the length of the wire,
B ,, bulk of the whole space occupied by wire and insulation,
n ,, ratio of this whole space to the bulk of the copper alone (that is, let $\frac{1}{n}B$ the bulk of the copper),
A ,, the sectional area of wire and insulator,
R ,, the resistance of the wire.

For the working coil, let the corresponding quantities be L', B', n', R'. Lastly, let s be the specific resistance of the copper. We have—

$$B = A L$$

$$R = ns\frac{L}{A} = ns\frac{B}{A^2}.$$

Hence,
$$A = \frac{\sqrt{(nsB)}}{\sqrt{R}} = \frac{K}{\sqrt{R}} \quad . \quad . \quad . \quad (1)$$

and similarly,
$$A' = \frac{\sqrt{(n's'B')}}{\sqrt{R'}} = \frac{K'}{\sqrt{R'}} \quad . \quad . \quad . \quad (2)$$

where K and K' denote constants.

Now, let c be the current through the magnet coil, and c' that through the working coil, and let v be the velocity of any chosen point of the working coil. Denoting by p the average electromotive force between the two ends of the working coil, we have—

$$p = I\frac{c}{A}\frac{1}{A'}v. \quad . \quad . \quad . \quad (3)$$

where I is a quantity depending on the forms, magnitudes, and relative positions of B and B', and on the magnetic susceptibility of iron; diminishing as the susceptibility diminishes with increased strength of current, or with any change of R and R' which gives increase of magnetising force.

In the single-circuit dynamo (that is, the ordinary dynamo) c' is equal to c, but not so in the shunt-dynamo. In each, the whole electric activity (that is, the rate of doing work) is pc'; or, by (3)—

$$I\frac{cc'}{AA'}v \quad . \quad . \quad . \quad . \quad (4)$$

or, by (1) and (2)—

$$\frac{I\sqrt{(RR')}cc'v}{KK'} \quad . \quad . \quad . \quad . \quad (5)$$

Of this whole work, the proportions which go to waste in heating the coils and to work in the external circuit are—

$$Rc^2 + R'c'^2 \quad . \quad . \text{ waste} \quad . \quad (6)$$

$$\frac{I\sqrt{(RR')}cc'v}{KK'} - (Rc^2 + R'c'^2). \text{ useful work} \quad (7)$$

By making v sufficiently great, the ratio of (6) to (7) (waste to useful work) may be made as small as we please. Our question is, how ought R and R' to be proportioned to make the ratio of waste to work a minimum, with any given speed? or, which comes to the same thing, to make the speed required for a given ratio of work to waste a minimum? To answer it, let r be the ratio of the whole work to the waste. We have, by (5) and (6)—

$$r = \frac{I\sqrt{(RR')}cc'}{Rc^2 + R'c'^2}\,\frac{v}{KK'} \quad . \quad . \quad (8)$$

For the single-circuit dynamo we have $c = c'$, and (8) becomes—

$$r = \frac{I\sqrt{(RR')}}{R + R'}\,\frac{v}{KK'} \quad . \quad . \quad . \quad (9)$$

or
$$r = \frac{I\sqrt{\{R(S-R)\}}v}{SKK'} \quad . \quad . \quad . \quad (10)$$

where
$$S = R + R' \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Suppose now S to be given, and suppose for a moment I to be constant. The problem of making r a maximum with v given, or v a minimum with r given, requires simply that $R(S-R)$ be a maximum; which it is when $R = \frac{1}{2}S$, that is, when the resistances in the working coil and the electro-magnet are equal. But in reality I is not constant; it diminishes with increase of the magnetising force. As it generally depends chiefly on the soft iron of the electro-magnet, and comparatively but little on the soft iron of the moving armature, or on iron magnetised by the current through the moving coils, it will generally be the case that I will, *cæteris paribus*, be diminished by increasing R and diminishing R'. Hence the maximum of r/v is shown by (10) to require R' to be somewhat greater than $\frac{1}{2}S$: how much greater we cannot find from the formula, without knowing the law of the variation of I.

Experience and natural selection seem to have led in most of the ordinary dynamos, as now made, to the resistance in the electro-magnet being somewhat less than the resistance in the working coil, which is in accordance with the preceding theory.

Whether the useful work of the dynamo be light-giving, or power, or heating, or electro-metallurgy, we may, for simplicity, reckon it in any possible case by referring to the convenient standard case of a current through a conductor of given resistance E connecting the working terminals of the dynamo. This conductor, in accordance with general usage, I call the "external circuit," which is an abbreviation for the part of the whole circuit which is external to the dynamo. In the case of the single-circuit dynamo the current in the external circle is equal to that through the working coil and electro-magnet, or c of our notation. Hence, by Ohm's law—

$$c = \frac{p}{E + R + R'} \quad . \quad . \quad . \quad . \quad (12)$$

or, by (3), (1), and (2),

$$c = c\frac{I\sqrt{(RR')}v}{KK'(E + R + R')} \quad . \quad . \quad (13)$$

Hence either
$$c = O \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

or
$$I = \frac{KK'(E + R + R')}{\sqrt{(RR')}v} \quad . \quad . \quad (15)$$

The case of $c = O$ is that in which

$$v < \frac{KK'(E + R + R'}{I_0\sqrt{(RR')}} \quad . \quad . \quad (16)$$

where I_0 denotes the value of I for $c = O$. To understand it, remember we are supposing no residual magnetism. For any speed subject to (16), the dynamo produces no current. When this limit is exceeded the electric equilibrium in the circuit becomes unstable; an infinitesimal current started in either direction rises rapidly in strength, till it is limited by equation (15), through the diminution of I, which it produces. Thus,

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[1] "On the Present Position and Prospects of Political Economy" (British Association, Dublin, 1878).

[2] Quoted by Mr. Grant Duff, Presidential Address to Section F, 1881.

[3] Paper read at the British Association, York, 1881, by Sir William Thomson, F.R.S.

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regarding I as a function of c, we have in (15) the equation mathematically expressing the strength of the current maintained by the dynamo when its regular action is reached. Using (15) in (10) we find—

$$r = \frac{E + S}{S} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

which we all knew forty years ago from Joule.

In the shunt-dynamo the whole current, c', of the working coil branches into two streams, c through the electro-magnet, and $c' - c$ through the external circuit, whose strengths are inversely as the resistances of their channels. Still calling the resistance of the external circuit E, we therefore have—

$$c R = (c' - c) E, \text{ which gives } c = \frac{E}{R + E} c' \quad . \quad (18)$$

Hence, by Joule's original law, the expenditures of work per unit of time in the three channels are respectively

$$\left.\begin{array}{ll} R' c'^2 \quad . \quad . \quad . \quad . & \text{working coil} \\ R\left(\frac{E}{R + E}\right)^2 c'^2 \quad . \quad . & \text{electro-magnet} \\ E\left(\frac{R}{R + E}\right)^2 c'^2 \quad . \quad . & \text{external circuit} \end{array}\right\} \quad . \quad (19)$$

Hence, denoting as above by r the ratio of the whole work to the work developed in the external circuit, we have—

$$r = \frac{R' + R\left(\frac{E}{R + E}\right)^2 + E\left(\frac{R}{R + E}\right)^2}{E\left(\frac{R}{R + E}\right)^2} \quad . \quad (20)$$

$$\left.\begin{array}{rl} \text{whence} \quad R^2 r & = R'\frac{(R + E)^2}{E} + R(R + E) \\ & = \frac{R' R^2}{E} + (R + R')E + R(2R' + R) \end{array}\right\} \quad (21)$$

Suppose now R and R' given, and E to be found; to make r a minimum. The solution is—

$$E = \sqrt{\frac{R' R^2}{R + R'}} \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and this makes

$$r = 2\sqrt{\frac{R'(R + R')}{R^2}} + \frac{2R' + R}{R} \quad . \quad . \quad (23)$$

Put now

$$\frac{R'}{R} = e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

(22) and (23) become

$$E = \sqrt{\frac{R R'}{1 + e}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$\text{and} \quad r = 1 + 2\sqrt{\{e(1 + e)\}} + 2e \quad . \quad . \quad . \quad (26)$$

For good economy r must be but little greater than unity; hence e must be very small, and therefore *approximately*

$$\left.\begin{array}{rl} & E = \sqrt{(R R')} \\ \text{and} & r = 1 + 2\sqrt{e} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

For example, suppose the resistance of the electro-magnet to be 400 times the resistance of the working coil—that is $e = 400$—and we have, approximately,

$$E = 20R', \text{ and } r = 1 + \tfrac{1}{10}.$$

That is to say, the resistance in the external circuit is twenty times the resistance of the working coil, and the useful work in the external circuit is approximately $\frac{10}{11}$ of that lost in heating the wire in the dynamo.

FUNCTIONAL METAMORPHOSIS OF MUSCLES[1]

THERE is no system in the animal body to which the axiom of Guérin, viz., that "function makes the organ," applies with greater force than to the muscular system. Every student of comparative myology knows that according to the use required of a muscle we have alterations in its volume and connections, or indeed its total disappearance, should its further services in the animal economy be dispensed with. These are the factors which render muscular homologies in many cases so difficult to determine. There is one change, however, which is much more common than is generally believed, viz., the transformation of a muscle into fibrous tissue, or, in other words, its replacement by a ligamentous structure possessing attachments similar to those of its muscular ancestor. It might almost be laid down as a law that whenever a muscle ceases to be of use for contractile purposes, and when, from its attachments, it might be of service as a ligament, that it gradually in course of time becomes transformed into fibrous tissue, and is handed down to posterity in this condition. Indeed should it merely be a case of comparative value, and should the balance of utility be in favour of a ligament, then also will this metamorphosis in all probability take place. Of all adaptations in the muscular system this is perhaps the most beautiful, and instances of it are by no means rare. Thus, in the feet of the armadillo, orycteropus, pig, walrus, and several other animals, certain of the intrinsic pedal muscles have become fibrous bands, indubitably retained for some definite purpose, although their obvious function is often obscure. The most striking examples of this, however, are to be found in the feet of the horse, ox, sheep, camel, and their allies. In these we are able not only to demonstrate with the utmost precision the particular muscles that have become ligamentous, but also the process by which the change has been brought about, and the *rationale* of the transformation.

The *suspensory ligament of the fetlock* in the horse is an exceedingly powerful structure, which lies in the sole of the foot (*i.e.* upon the posterior aspect of the metatarsal bone) under cover of the flexor tendons. It plays an important part in the mechanism of the limb. Its attachments are such that it prevents over extension at the fetlock or metatarso-phalangeal joint, and its value in this respect is evidenced by the fact that when it is ruptured the horse becomes what is termed by veterinary surgeons "broken down." In this condition the fetlock joint sinks downwards towards the ground, whilst the hoof is tilted forwards and upwards.

This ligament is admitted on all hands to be derived from the intrinsic pedal muscles by a transformation of the muscular elements into fibrous tissue; indeed it bears its history written upon its face. Almost invariably a narrow streak of striated muscular fibres can be detected upon its superficial surface which points to its muscular origin. Upon its deep surface fleshy fibres in greater abundance are observed, but these are very pale, owing to a large admixture of fatty tissue.

The question now comes to be—Which of the intrinsic pedal muscles have entered into the formation of this ligament? In making this inquiry we have to keep two points in view: (1) that in the horse the middle or third digit is alone fully developed; and (2) that in a typical pentadactylous foot this digit is supplied by three intrinsic muscles, viz.: a two-headed flexor brevis, and two abductors or dorsal interossei (the second and third) inserted one upon either side of the digit. It is reasonable to conclude, therefore, that the suspensory ligament of the fetlock is derived from one or more of these muscles. But independent remnants of the two dorsal interossei are present, in addition to the ligament, which clearly proves the *flexor brevis medii* to be the source of this structure.

The dorsal interossei in the foot of the horse are of peculiar interest. They are so minute that they can exercise little or no influence upon the movements of the pes. They are simply to be regarded as vestiges of former greatness, and as pointing to retrograde development. They undoubtedly constitute a link in the soft parts between our modern monodactylous horse and its three-toed ancestor. They lead us back to a time when in the foot of this animal there were two distinct interosseous spaces, each filled by a well-marked interosseous muscle.

Still stronger evidence that the suspensory ligament originates solely from the *flexor brevis* of the middle digit is obtained by making thin transverse sections through its substance. We then observe that the sparse remains of muscular tissue are not confined to the surface of the structure, but penetrate into its midst. When the specimen is held against a dark ground, two crescentic opaque outlines are noticed lying side by side in its substance. These undoubtedly represent tracings of the two heads of the flexor brevis, out of which the ligament is developed. On subjecting the outlines to microscopic examination, we find that they are mainly composed of muscular fibres, but every here and there the continuity of this tissue is broken by fatty tissue, in which are observed transversely-divided nerves and blood-vessels.

In the ox, sheep, and camel the suspensory ligament performs the same office as in the horse. The presence, however, of two digits (the middle and annular) complicates somewhat its inferior attachments, in order that it may operate so as to prevent over-extension at both metatarso-phalangeal joints. In each of these animals the structure is undoubtedly formed by the two heads of

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[1] Abstract of paper read at the York meeting of the British Association, by D. J. Cunningham, M.D., F.R.S.E., Senior Demonstrator of Anatomy, University of Edinburgh.

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